International Information Technology University JSC
Faculty of Information Technology
Department of Mathematical and Computer Modeling

Approved
Vice-Rector of Academic and Educational
Affairs of IITU JSC, PhD
Umarov T.F.
«28» 08 2020

# SYLLABUS
# (ACADEMIC PROGRAM)

**Course:** TV 2214 Probability theory

**Major**: 6B061 Information and communication technology

**Educational program** 6B06111 Financial mathematics,
6B06112 Data Science

**Year**: 2 **Semester:** 3 **Number of credits**: 4 ECTS

**Lectures:** 15 hours

**Practical classes:** 30 hours

**T/SIS:** 75 hours

**Total:** 120 hours

**Final assessment form**: Examination

Almaty 2020

Academic program of the course "TV 2214 Probability theory" has been reviewed a the meeting of the Mathematical and Computer Modeling department.

Minutes №. 1 dated «17»August 2020

Head of the Department,
PhD,
Assistant-Professor _____ Ydyrys A.Zh.

Authors:

Assistant professor,
PhD Даркенбаева Г.С Darkenbayeva G.S.

The working academic program was approved at the meeting of the Educational and Methodological Board of JSC "IITU"

Minutes № 1 dated "28" August 2020 .

Director of the Department _____ A.Mustafina
*Signature*
for Academic Affairs

| 1. GENERAL INFORMATION | |
|---|---|
| Faculty | Information Technology |
| Major code and title | 6B061 Information and communication technology |
| Educational program code and title | 6B06111 Financial mathematics, 6B06112 Data Science |
| Year, semester | 2nd year, 3rd semester |
| Subject category | Basic |
| Number of credits (ECTS) | 4 ECTS |
| Language of Delivery | English |
| Prerequisites | Mathematical analysis, Linear Algebra and Analytical Geometry, Discrete Mathematics |
| Post Requisites | Number theory, Data science: Introduction to machine Learning, Bayesian statistics and analysis, Statistics 1 |

**2. GOALS, OBJECTIVES AND LEARNING OUTCOMES OF THE COURSE**

**The course goal is** at the end of the course Probability theory, the students should be acquainted with basic principles of probability theory and statistics, should have experience in carrying out of experimental works, in critical thinking and with techniques solving problems.

**The objectives of the course are**

- to provide the student with a basic knowledge of statistics: sampling method, methods of estimations of parameters, testing statistical hypotheses, correlation and regression analysis, dispersion analysis.
- to provide the student with a sufficient background in these areas so that the student will then be ready to take advanced courses in these areas.
- to provide the student with the knowledge of these areas necessary to the pursuit of his/her major course of study in science or engineering.
- to develop in the student an analytic approach to problem solving, both in science and "everyday life".

to develop in the student an appreciation of the role of science in our current society, as well as in the past, and towards the future.

**Learning outcomes of the course**

**By the end of this course the students will be able to:**

- demonstrate mastery of mathematical concepts, analysis and techniques to the advanced level.
- apply concepts and techniques from mathematics and statistics to economic problems.
- obtain a deep insight into Probability and Statistics in the context of advanced mathematical frameworks, independently study special mathematical literature.
- use computer technologies and techniques.
- think clearly, sequentially, and logically, as required for critical analysis of quantitative problems, computer problems.
- process and evaluate effectively both theoretical and real-life quantitative data.
- select suitable mathematical methods and algorithms for the solution of a problem.

carry out qualitative mathematical research.

**3. Course description**

The course Probability Theory and Mathematical Statistics focuses on the relationship between mathematics and programming, operation systems through an interdisciplinary program of study covering the cross-section of mathematical analysis, modern statistical techniques and economic theory which deepens the students' mathematical understanding of probability, practical market issues and develops their logical and algorithmic thinking skills. The course provides students with the mathematical tools to be applied to computer problems, advanced methods of studying and solving mathematical problems. Strong backgrounds in calculus, linear algebra are recommended.

**4. COURSE POLICY**

Students are not allowed to miss classes and use cell phones during classes. There are no late workshops for students who missed lecture or labor class. (NO "OTRABOTKA") Instructor may change course outline at any time during the course. Students may come to see Tutor only at Office Hours' time or by appointment.
Deadline for each task must 2 weeks.

**Plagiarism**

Plagiarism is the copying or close paraphrasing of published or unpublished work, including the work of another student, without the use of quotation marks and due acknowledgement.
Plagiarism and collusion are offences, and the University takes cases of such offences very seriously. Students found guilty of committing either offence will be punished.
You should not lend your work to another student, even if that student is a friend and you are trying to be helpful. If your friend's work appears to be similar to yours, you both run the risk of being penalized and get mark ZERO.

**Control work:**

During control works the teacher provides samples and tasks, which should be in line with the learned material. The duration of the control work depends on the number of samples and tasks, but it should not exceed 50 minutes.
At the end of each section, the teacher conducts control works based on theoretical issues. The teacher has a right to include themes, which were provided to the students for independent learning.
The duration of control works is typically 30-50 minutes, and the teacher provides 5-8 questions.
The student, which was seen with cheat sheets or misconduct during control work, will be expelled from class without a right to re-take the control work.
Final exam: At the end of the course the students will pass final exam, which is evaluated as 40 points (maximum).

**Additional remarks:**

- Attendance (always be in time on lectures)
- Read main and additional materials
- Do homework

**Online classes** will be on MS Teams, DL and Zoom

## 5. LITERATURE

Basic literature

1. Marco Taboga, "Lectures on probability theory and mathematical statistics", eighth edition, University of Southern California, Pearson Education International, 2017, 561 pages.
2. Prasanna Sahoo, "Probability theory and mathematical statistics", third edition, University of Loisville, Elsevier, 2016, 818 pages.
3. https://www.coursera.org/programs/international-information-technologies-university-on-coursera-x7m8z/browse?=&collectionId=&productId=Qj2TzfF1EemW0hLBeDiHeA&productType=course&query=Object+Oriented&showMiniModal=true

Supplementary literature:

3. Gmurman V.E., Probability theory and mathematical statistics. The seventh issue, stereotypical, Moscow: «Higher school», 2000 (in Russian).
4. Gorelova G.V., Katsko I.A., Probability theory and mathematical statistics in examples and exercises with using Excel. – The fourth issue, Rostov upon Don: «Fenix», 2006 (in Russian).
5. Cremer N.Sh., Probability theory and mathematical statistics: Manual for higher educational organizations. — The second edition, reprocessed and added. M: UNITY-DANA, 2004. — 573 p. (in Russian)
6. Danko P.E., Popov A.G., Kozhevnikova T.Ya., Higher mathematics in exercises. The fifth issue, revised. – Moscow: «Higher school», 1999, part 2 (in Russian).
7. Olav Kallenberg, Foundations of Modern Probability, second edition, Springer Series in Statistics, 2002, 650 pages.

**6. Course schedule**

| Week No | Course Topic | Reference Materials | Lectures (1 h/w) | Practical classes (2 h/w) | TSIS (1 h/w) | SIS (4 h/w) |
|---|---|---|---|---|---|---|
| 1 | Introduction. Basic concepts of probability theory. Classical definition of probability. Relative frequency. Geometrical probabilities. | Basic [1], [2] | L 1 | PA 1 | TSIS 1 | SIS 1 |
| 2 | Basic formulas of combinatorial analysis. Operations over events. | Basic [1], [2] | L 2 | PA 2 | TSIS 2 | SIS 2 |
| 3 | Theorem of addition of probabilities of incompatible events. Complete group of events. Opposite events. Conditional probability. | Basic [1]-[3] | L 3 | PA 3 | Quiz 1 | |
| 4 | Independent events. Probability of appearance of at least one event. | Basic [1], [2] | L 4 | PA 4 | TSIS 3 | SIS 3 |
| 5 | Theorem of addition of probabilities of compatible events. Formula of total probability. Probability of hypotheses. Bayes's formulas. | Basic [1]-[3] | L 5 | PA 5 | TSIS 4 | SIS 4 |
| 6 | Repetition (recurrence) of trials. The Bernoulli formula. Local theorem of Laplace. Integral theorem of Laplace. | Basic [1]-[3] | L 6 | PA 6 | Quiz 2 | |
| 7 | Random variables. The law of distribution of a discrete random variable. Mathematical operations over random variables. (Mathematical) expectation of a discrete random variable. Dispersion of a discrete random variable. | Basic [1]-[3] | L 7 | PA 7 | TSIS 5 | SIS 5 |
| 8 | Distribution function of a random variable. Continuous random variables. Probability density. | Basic [1]-[3] | L 8 | PA 8 | Mid-term | SIS 6 |
| 9 | Basic laws of distribution of discrete random variables. | Basic [1]-[3] | L 9 | PA 7 | TSIS 6 | SIS 7 |
| 10 | Basic laws of distribution of continuous random variables | Basic [1]-[3] | L 10 | PA 10 | TSIS 7 | SIS 8 |
| 11 | The law of large numbers and limit theorems. | Basic [1]-[3] | L 11 | PA 11 | Quiz 3 | |
| 12 | Introduction to Statistics. Inferential Statistics and Probability Models. Population and Samples. | Basic [1]-[3] | L 12 | PA 12 | TSIS 8 | SIS 9 |
| 13 | Describing Data Sets. Frequency tables and graphs. Grouped data and histograms. | Basic [1]-[3] | L 13 | PA 13 | Quiz 4 | |
| 14 | Using Statistics to summarize data sets. Sample mean. Sample | Basic [1]-[3] | L 14 | PA 14 | TSIS 9 | SIS 10 |

| | | | | | | |
|---|---|---|---|---|---|---|
| | Median. Sample Mode. Sample Variance and Sample Standard Deviation. | | | | | |
| 15 | Testing Statistics Hypothesis. Hypothesis Tests and Significance levels. Tests concerning the Mean of a Normal Population: Cases of known and unknown variance. | Basic [1]-[3] | L 15 | PA 15 | End of term | |
| | **Total hours** | **120** | **15** | **30** | **15** | **60** |

**7. List of topics/ assignments for practical classes**

| № | Topic Title | Number of hours | References | Form of reporting | Deadline |
|---|---|---|---|---|---|
| 1 | Introduction. Basic concepts of probability theory. Classical definition of probability. Relative frequency. Geometrical probabilities. | 2 | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | Week 1 |
| 2 | Basic formulas of combinatorial analysis. Operations over events. | 2 | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | Week 2 |
| 3 | Theorem of addition of probabilities of incompatible events. Complete group of events. Opposite events. Conditional probability. | 2 | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | Week 3 |
| 4 | Independent events. Probability of appearance of at least one event. | 2 | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | Week 4 |
| 5 | Theorem of addition of probabilities of compatible events. Formula of total probability. Probability of hypotheses. Bayes's formulas. | 2 | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | Week 5 |
| 6 | Repetition (recurrence) of trials. The Bernoulli formula. Local theorem of Laplace. Integral theorem of Laplace. | 2 | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | Week 6 |
| 7 | Random variables. The law of distribution of a discrete random variable. Mathematical operations over random variables. (Mathematical) expectation of a discrete random variable. Dispersion of a discrete random variable. | 2 | Basic [1]-[3] | L 7 | Week 7 |

| | | | | | |
|---|---|---|---|---|---|
| 8 | Distribution function of a random variable. Continuous random variables. Probability density. | 2 | Basic [1]-[3] | L 8 | Week 8 |
| 9 | Basic laws of distribution of discrete random variables. | 2 | Basic [1]-[3] | L 9 | Week 9 |
| 10 | Basic laws of distribution of continuous random variables | 2 | Basic [1]-[3] | L 10 | Week 10 |
| 11 | The law of large numbers and limit theorems. | 2 | Basic [1]-[3] | L 11 | Week 11 |
| 12 | Introduction to Statistics. Inferential Statistics and Probability Models. Population and Samples. | 2 | Basic [1]-[3] | L 12 | Week 12 |
| 13 | Describing Data Sets. Frequency tables and graphs. Grouped data and histograms. | 2 | Basic [1]-[3] | L 13 | Week 13 |
| 14 | Using Statistics to summarize data sets. Sample mean. Sample Median. Sample Mode. Sample Variance and Sample Standard Deviation. | 2 | Basic [1]-[3] | L 14 | Week 14 |
| 15 | Testing Statistics Hypothesis. Hypothesis Tests and Significance levels. Tests concerning the Mean of a Normal Population: Cases of known and unknown variance. | 2 | Basic [1]-[3] | L 15 | Week 15 |

## 8. List of topics/assignments for Student Independent Study

Proper organization of students independent study is the key to the formation of skills in mastering, learning, assimilation and systematization of acquired knowledge, ensuring a high level of academic performance in the learning process

| № | Assignments (topics) for Independent study | Recommended literature and other sources (links) | Form of submission | № | Assignments (topics) for Independent study | Deadline |
|---|---|---|---|---|---|---|
| 1 | Foundations of Probability Theory | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | 1 | Foundations of Probability Theory | Week 2 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | Discrete random variables. | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | 2 | Discrete random variables. | Week 4 |
| 3 | Continuous random variables. | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | 3 | Continuous random variables. | Week 6 |
| 4 | Multivariate random variables. | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | 4 | Multivariate random variables. | Week 8 |
| 5 | Convergence. | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | 5 | Convergence. | Week 10 |
| 6 | Properties of estimators and maximum likelihood estimation. | Basic [1]-[3] Supplementary [1][2] | Submission of IHW | 6 | Properties of estimators and maximum likelihood estimation. | Week 12 |

**9. System for evaluating student performance in a discipline:**

| Period | Assignments | Number of points | Total |
|---|---|---|---|
| 1st attestation | **Class work:**<br>Regular attendance\Active participation<br>Quiz # 1<br>Quiz # 2<br>**Student Independent Study:**<br>Individual Home Work from #1 to #5<br>Individual Home Work # 6<br>**Mid term** | **40**<br>10<br>15<br>15<br>**35**<br>5*5=25<br>10<br>**25** | 100 |
| 2nd attestation | **Class work:**<br>Regular attendance\Active participation<br>Quiz # 3<br>Quiz # 4<br>**Student Independent Study:**<br>Individual Home Work from #7 to #13<br>**End of term** | **40**<br>10<br>15<br>15<br>**35**<br>5*7=35<br>**25** | 100 |
| Final exam | | | **Exam** |
| **Total** | **0,3*1stAtt+0,3*2ndAtt+0,4*Final** | | |

*If the number of absences exceeds 20%, student will be automatically scheduled for a Retake (summer semester)

**10. Assessment criteria:**

The point-rating letter system for assessing the educational achievements of students with their interpretation in the traditional grading scale:

| Letter Grade | Numerical equivalent | Points (%) | Traditional system assessment | General description of grading criteria |
|---|---|---|---|---|
| A | 4,0 | 95-100 | Excellent | The student has knowledge of the subject in the full scope of the curriculum, understands the discipline deeply enough; shows a high level of knowledge that exceeds the volume provided by the syllabus, gives an exhaustive answer |
| A- | 3,67 | 90-94 | | The student has knowledge of the subject in the full scope of the curriculum, understands the discipline deeply enough; gives an exhaustive answer |
| B+ | 3,33 | 85-89 | Good | The student shows a complete, well-founded knowledge of the subject, but the answers did not always highlight the main idea, rational methods of calculation were not always used; the answers were mostly brief and sometimes unclear. |
| B | 3,0 | 80-84 | | |
| B- | 2,67 | 75-79 | | |
| C+ | 2,33 | 70-74 | | |
| C | 2,0 | 65-69 | Satisfactory | The student demonstrates sufficient knowledge of the subject, but without proper depth and justification, the answers are unclear and without proper logical sequence. |
| C- | 1,67 | 60-64 | | |
| D+ | 1,33 | 55-59 | | |
| D | 1,0 | 50-54 | | |
| FX | 0,5 | 25-49 | Unsatisfactory | The student demonstrates insufficient knowledge of the subject, positive answers were not given to individual questions. |
| F | 0 | 0-24 | | The student demonstrates a very low level of knowledge of the subject. |

## 11. Assessment and evaluation materials (exam questions)

Questions for mid-term:

Basic concepts of probability theory. Classical definition of probability. Relative frequency. Geometrical probabilities. Basic formulas of combinatorial analysis. Operations over events. Theorem of addition of probabilities of incompatible events. Complete group of events. Opposite events. Conditional probability. Independent events. Probability of appearance of at least one event. Theorem of addition of probabilities of compatible events. Formula of total probability. Probability of hypotheses. Bayes's formulas. Repetition (recurrence) of trials. The Bernoulli formula. Local theorem of Laplace. Integral theorem of Laplace. Random variables. The law of distribution of a discrete random variable. Mathematical operations over random variables. (Mathematical)

expectation of a discrete random variable. Dispersion of a discrete random variable. Distribution function of a random variable. Continuous random variables. Probability density.

Questions for end of term:

Basic laws of distribution of discrete random variables. Basic laws of distribution of continuous random variables. The law of large numbers and limit theorems. Inferential Statistics and Probability Models. Population and Samples. Describing Data Sets. Frequency tables and graphs. Grouped data and histograms. Using Statistics to summarize data sets. Sample mean. Sample Median. Sample Mode. Sample Variance and Sample Standard Deviation. Testing Statistics Hypothesis. Hypothesis Tests and Significance levels. Tests concerning the Mean of a Normal Population: Cases of known and unknown variance.

Questions for final examination:

Basic concepts of probability theory. Classical definition of probability. Relative frequency. Geometrical probabilities. Basic formulas of combinatorial analysis. Operations over events. Theorem of addition of probabilities of incompatible events. Complete group of events. Opposite events. Conditional probability. Independent events. Probability of appearance of at least one event. Theorem of addition of probabilities of compatible events. Formula of total probability. Probability of hypotheses. Bayes's formulas. Repetition (recurrence) of trials. The Bernoulli formula. Local theorem of Laplace. Integral theorem of Laplace. Random variables. The law of distribution of a discrete random variable. Mathematical operations over random variables. (Mathematical) expectation of a discrete random variable. Dispersion of a discrete random variable. Distribution function of a random variable. Continuous random variables. Probability density. Basic laws of distribution of discrete random variables. Basic laws of distribution of continuous random variables. The law of large numbers and limit theorems. Inferential Statistics and Probability Models. Population and Samples. Describing Data Sets. Frequency tables and graphs. Grouped data and histograms. Using Statistics to summarize data sets. Sample mean. Sample Median. Sample Mode. Sample Variance and Sample Standard Deviation. Testing Statistics Hypothesis. Hypothesis Tests and Significance levels. Tests concerning the Mean of a Normal Population: Cases of known and unknown variance.

The quizzes, mid-term will be in the form of a **test**, the end of term and **final examination** will be in the form of a **test**.

Sample Exam card with assessment criteria:

1.) Students pass a test if they score 50% or more. The marks of a large number of students were sampled and the mean and standard deviation were calculated as 42% and 8% respectively. Assuming this data is normally distributed, what percentage of students pass the test?

a.) 28%; b.) 16%; c.) 5%; d.) 24%; e.) 35%.

2.) The Fresha Tea Company pack tea in bags marked as 250 g. A large number of packs of tea were weighed and the mean and standard deviation were calculated as 255 g and 2.5 g respectively. Assuming this data is normally distributed, what percentage of packs are underweight?

a.) 2%; b.) 3%; c.) 2.5%; d.) 5%; e.) 3.5%.

3.) Miss Jones has 30 students in her math class. In the recent exam, her students averaged 67%. But Miss Jones told Principal Schultz that her students had averaged 71%. In order to impress Principal Schultz, Miss Jones had excluded the two outliers in her class who had scored very low marks. What was the mean mark of those two *'poor'* students *(those who got low marks)*?

a.) 11%; b.) 9%; c.) 12%; d.) 10%; e.) 7%.

4.) 50 children guessed the number of marbles in a jar and the average guess was 627. However, three of the guesses were way too high and so were excluded from the competition. When these three outliers were excluded, the average guess was reduced by 114. What was the mean of the three outliers?

a.) 3.485; b.) 2.964 c.) 4.642; d.) 2.413; e.) 1.539.

5.) From past experience a professor knows that the test score of a student taking her final examination is a random variable with mean 75. Suppose, in addition, the professor knows that a variance of a student's test score is equal to 25. Find the right inequation.

a.) $P(|X-75|<30)>\frac{1}{36}$, b.) $P(65\le X\le 85)\ge\frac{3}{4}$ $c.)$ $P(60\le X\le 90)\ge\frac{1}{9}$,

d.)$P(X>80)>\frac{75}{80}$, e.) $P(|X-75|\le 45)<\frac{80}{81}$

Assessment criteria

| Letter Grade | Numerical equivalent | Points (%) | Traditional system assessment |
|---|---|---|---|
| A | ▪ 4,0 | 95-100 | ▪ Excellent |
| A- | 3,67 | 90-94 | |
| B+ | 3,33 | 85-89 | Good |
| B | 3,0 | 80-84 | |
| B- | 2,67 | 75-79 | |
| C+ | 2,33 | 70-74 | |
| C | 2,0 | 65-69 | Satisfactory |
| C- | 1,67 | 60-64 | |
| D+ | 1,33 | 55-59 | |
| D | 1,0 | 50-54 | |
| FX | 0,5 | 25-49 | Unsatisfactory |
| F | 0 | 0-24 | |